FORMULA REFERENCE

Z-Score Formulas

Match the score, mean, and standard deviation to the same reference group, and use a normal model only when the context supports it.

Author: Mathos AI
Publisher: Mathos AI | https://www.mathos.ai/
Generated on: 2026-09-11

Standardize a value

Express a raw value as a signed number of standard deviations from its reference mean.

Population z-score

$$z = \frac{x - \mu}{\sigma}$$

Text version: Z equals x minus mu divided by sigma.

When to use: Use a population z-score to measure how many population standard deviations a value lies above or below the mean.

Conditions

- Sigma is the population standard deviation and is greater than zero.
- x and mu use the same units.

Sample-standardized score

$$z = \frac{x - \bar{x}}{s}$$

Text version: A sample-standardized score z equals x minus the sample mean x bar divided by the sample standard deviation s.

When to use: Use this descriptive standardization to locate a value relative to the mean and standard deviation of the observed sample.

Conditions

- s is greater than zero.
- x, x bar, and s refer to the same sample and use matched units.

Reverse and compare

Recover a raw value or compare measurements recorded on different scales.

Recover a raw value

$$x = \mu + z\sigma$$

Text version: The raw value x equals the population mean mu plus z times the population standard deviation sigma.

When to use: Use this rearrangement to recover a raw value from a population z-score and its original reference distribution.

Conditions

- Mu and sigma are the same reference values used to define z.
- Sigma is greater than zero.

Compare values from different scales

$$z_1 > z_2 \iff \frac{x_1 - \mu_1}{\sigma_1} > \frac{x_2 - \mu_2}{\sigma_2}$$

Text version: Z one is greater than z two exactly when the standardized value from the first scale is greater than the standardized value from the second scale.

When to use: Use matched z-scores to compare relative positions from different populations or measurement scales.

Conditions

- Each standard deviation is greater than zero.
- Each value is matched to its own relevant reference mean and standard deviation.

Normal-model probability

Connect a standardized score to cumulative probability only under a justified normal model.

Normal cumulative probability from a z-score

$$P(X \le x) = \Phi\left(\frac{x - \mu}{\sigma}\right) = \Phi(z)$$

Text version: For a normal random variable X, the probability that X is at most x equals the standard normal cumulative distribution function Phi evaluated at z.

When to use: Use this relation to convert a z-score to a lower-tail probability when a normal distribution model is justified.

Conditions

- X follows a normal distribution with mean mu and positive standard deviation sigma.
- Phi is the cumulative distribution function of the standard normal distribution.