FORMULA REFERENCE

Statistics Formulas

Choose a formula by task, identify whether the data are a sample or population, and confirm every condition.

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Descriptive statistics

Summarize the center and spread of a numerical dataset.

Arithmetic mean

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

Text version: X bar equals one divided by n times the sum from i equals one to n of x sub i.

When to use: Use the arithmetic mean to describe the center of numerical observations when each observation has equal weight.

Conditions

- n is a positive integer.
- The data contain n numerical observations.

Weighted mean

$$\bar{x}_w = \frac{\sum_{i=1}^{n} w_i x_i}{\sum_{i=1}^{n} w_i}$$

Text version: The weighted mean x bar sub w equals the sum of w sub i times x sub i divided by the sum of w sub i.

When to use: Use the weighted mean when observations contribute different known weights.

Conditions

- n is a positive integer.
- Weights are nonnegative and at least one weight is positive.
- Each weight is paired with its corresponding observation.

Range

$$R = x_{\max} - x_{\min}$$

Text version: The range R equals the maximum observation minus the minimum observation.

When to use: Use the range for a quick description of the full spread from the smallest to the largest observation.

Conditions

- The dataset is nonempty.
- Maximum and minimum are evaluated in the same units.

Interquartile range

$$\mathrm{IQR} = Q_3 - Q_1$$

Text version: The interquartile range equals the third quartile minus the first quartile.

When to use: Use the interquartile range to describe the spread of the middle half of a dataset.

Conditions

- Q sub one and Q sub three are computed from the same dataset.
- Use one stated quartile convention consistently because conventions can differ.

Variation and standard scores

Distinguish population and sample spread, then standardize observations when appropriate.

Population variance

$$\sigma^2 = \frac{1}{N}\sum_{i=1}^{N}(x_i - \mu)^2$$

Text version: Sigma squared equals one divided by capital N times the sum from i equals one to capital N of x sub i minus mu, squared.

When to use: Use population variance when the values represent the complete population being described.

Conditions

- N is a positive integer.
- Mu is the arithmetic mean of the N population values.

Population standard deviation

$$\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - \mu)^2}$$

Text version: Sigma equals the square root of one divided by capital N times the sum of squared deviations of x sub i from mu.

When to use: Use population standard deviation to express population spread in the original data units.

Conditions

- N is a positive integer.
- Mu is the population mean.
- Use the nonnegative square root.

Sample variance

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

Text version: S squared equals one divided by n minus one times the sum from i equals one to n of x sub i minus x bar, squared.

When to use: Use this sample-variance definition when estimating spread from a sample with Bessel's correction.

Conditions

- n is at least two.
- X bar is the arithmetic mean of the n sample observations.

Sample standard deviation

$$s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

Text version: S equals the square root of one divided by n minus one times the sum of squared deviations of x sub i from x bar.

When to use: Use sample standard deviation to express sample spread in the original data units.

Conditions

- n is at least two.
- X bar is the sample mean.
- Use the nonnegative square root.

Population z-score

$$z = \frac{x - \mu}{\sigma}$$

Text version: Z equals x minus mu divided by sigma.

When to use: Use a population z-score to measure how many population standard deviations a value lies above or below the mean.

Conditions

- Sigma is the population standard deviation and is greater than zero.
- x and mu use the same units.

Probability

Combine events, condition on known information, and model repeated independent trials.

Complement rule

$$P(A^c) = 1 - P(A)$$

Text version: The probability of the complement of event A equals one minus the probability of event A.

When to use: Use the complement rule when the probability of an event is easier to find from the probability that it does not occur.

Conditions

- A is an event in a probability space.
- A superscript c contains exactly the outcomes not in A.

General addition rule

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Text version: The probability of A union B equals the probability of A plus the probability of B minus the probability of A intersection B.

When to use: Use the general addition rule to find the probability that at least one of two events occurs.

Conditions

- A and B are events in the same probability space.

Conditional probability

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

Text version: The probability of A given B equals the probability of A intersection B divided by the probability of B.

When to use: Use conditional probability when the sample space is restricted to outcomes in event B.

Conditions

- A and B are events in the same probability space.
- The probability of B is greater than zero.

Multiplication rule

$$P(A \cap B) = P(A \mid B)P(B)$$

Text version: The probability of A intersection B equals the probability of A given B times the probability of B.

When to use: Use the multiplication rule to find a joint probability from a conditional probability and its conditioning event.

Conditions

- A and B are events in the same probability space.
- The probability of B is greater than zero for the conditional term shown.

Binomial probability

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$$

Text version: The probability that X equals k is n choose k times p to the k times one minus p to the n minus k.

When to use: Use the binomial formula for exactly k successes in n independent trials with one constant success probability.

Conditions

- n is a nonnegative integer and k is an integer from zero through n.
- Trials are independent.
- Each trial has two outcomes and the same success probability p, where p is between zero and one inclusive.