FORMULA REFERENCE

Standard Deviation Formulas

Decide whether the data represent a population or a sample before choosing the denominator, then keep the result in the original units.

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Population spread

Describe the full population with the population mean and denominator N.

Population variance

$$\sigma^2 = \frac{1}{N}\sum_{i=1}^{N}(x_i - \mu)^2$$

Text version: Sigma squared equals one divided by capital N times the sum from i equals one to capital N of x sub i minus mu, squared.

When to use: Use population variance when the values represent the complete population being described.

Conditions

- N is a positive integer.
- Mu is the arithmetic mean of the N population values.

Population standard deviation

$$\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - \mu)^2}$$

Text version: Sigma equals the square root of one divided by capital N times the sum of squared deviations of x sub i from mu.

When to use: Use population standard deviation to express population spread in the original data units.

Conditions

- N is a positive integer.
- Mu is the population mean.
- Use the nonnegative square root.

Population variance computational form

$$\sigma^2 = \frac{1}{N}\sum_{i=1}^{N} x_i^2 - \mu^2$$

Text version: Population variance equals the mean of the squared values minus the square of the population mean.

When to use: Use this identity as an independent arithmetic check for population variance while retaining sufficient precision.

Conditions

- The N values form the full population being described.
- Mu is the population mean of the same values.

Sample spread

Estimate population spread from a sample with the n minus 1 denominator.

Sample variance

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

Text version: S squared equals one divided by n minus one times the sum from i equals one to n of x sub i minus x bar, squared.

When to use: Use this sample-variance definition when estimating spread from a sample with Bessel's correction.

Conditions

- n is at least two.
- X bar is the arithmetic mean of the n sample observations.

Sample standard deviation

$$s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

Text version: S equals the square root of one divided by n minus one times the sum of squared deviations of x sub i from x bar.

When to use: Use sample standard deviation to express sample spread in the original data units.

Conditions

- n is at least two.
- X bar is the sample mean.
- Use the nonnegative square root.

Sample variance computational form

$$s^2 = \frac{\sum_{i=1}^{n} x_i^2 - n\bar{x}^{\,2}}{n-1}$$

Text version: Sample variance equals the sum of squared values minus n times x bar squared, divided by n minus one.

When to use: Use this identity to check a sample-variance calculation while keeping exact or high-precision intermediate values.

Conditions

- n is at least two.
- X bar is the sample mean of the same n observations.

Changes of scale and location

A shift leaves standard deviation unchanged, while scaling changes it by the absolute scale factor.

Standard deviation under a linear transformation

$$\mathrm{SD}(a + bX) = |b|\,\mathrm{SD}(X)$$

Text version: The standard deviation of a plus b times X equals the absolute value of b times the standard deviation of X.

When to use: Use this rule when every observation is shifted by a and scaled by b.

Conditions

- a and b are constants.
- The standard deviation of X is finite.