

Probability Formulas

Define the events first, then choose a formula that matches the stated relationship instead of assuming independence.

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Counting equally likely outcomes

Count the sample space only when every listed outcome has the same probability.

Equally likely outcomes

$$P(A) = \frac{|A|}{|S|}$$

Text version: The probability of event A equals the number of outcomes in A divided by the number of outcomes in sample space S.

When to use: Use this counting formula only for a finite sample space in which every elementary outcome is equally likely.

Conditions

- S is finite and nonempty.
- Every elementary outcome in S is equally likely.
- A is a subset of S.

Combination count

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}$$

Text version: N choose r equals n factorial divided by r factorial times n minus r factorial.

When to use: Use combinations to count selections of r objects from n distinct objects when order does not matter.

Conditions

- n is a nonnegative integer.
- r is an integer from zero through n.
- Order does not matter.

Complements and combined events

Use complements, unions, and intersections without double-counting overlap.

Complement rule

$$P(A^c) = 1 - P(A)$$

Text version: The probability of the complement of event A equals one minus the probability of event A.

When to use: Use the complement rule when the probability of an event is easier to find from the probability that it does not occur.

Conditions

- A is an event in a probability space.
- A superscript c contains exactly the outcomes not in A.

General addition rule

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Text version: The probability of A union B equals the probability of A plus the probability of B minus the probability of A intersection B.

When to use: Use the general addition rule to find the probability that at least one of two events occurs.

Conditions

- A and B are events in the same probability space.

Conditional probability

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

Text version: The probability of A given B equals the probability of A intersection B divided by the probability of B.

When to use: Use conditional probability when the sample space is restricted to outcomes in event B.

Conditions

- A and B are events in the same probability space.
- The probability of B is greater than zero.

Multiplication rule

$$P(A \cap B) = P(A | B)P(B)$$

Text version: The probability of A intersection B equals the probability of A given B times the probability of B.

When to use: Use the multiplication rule to find a joint probability from a conditional probability and its conditioning event.

Conditions

- A and B are events in the same probability space.
- The probability of B is greater than zero for the conditional term shown.

Independent-event multiplication

$$P(A \cap B) = P(A)P(B)$$

Text version: For independent events A and B, the probability of their intersection equals the probability of A times the probability of B.

When to use: Use this shorter multiplication rule only after the model states or establishes that A and B are independent.

Conditions

- A and B are independent events in the same probability space.

Repeated independent trials

Use the binomial model only when the trial count and success probability are fixed.

Binomial probability

$$P(X = k) = \binom{n}{k} p^k (1 - p)^{n-k}$$

Text version: The probability that X equals k is n choose k times p to the k times one minus p to the n minus k.

When to use: Use the binomial formula for exactly k successes in n independent trials with one constant success probability.

Conditions

- n is a nonnegative integer and k is an integer from zero through n.
- Trials are independent.
- Each trial has two outcomes and the same success probability p, where p is between zero and one inclusive.