FORMULA REFERENCE

Limit Formulas

Use limit laws only after checking their hypotheses, and treat one-sided behavior separately when the sides can differ.

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Limit laws

Combine component limits when each required limit exists and every denominator stays nonzero.

Constant limit law

$$\lim_{x \to a} c = c$$

Text version: The limit as x approaches a of the constant c equals c.

When to use: Use this law for a term that does not depend on the approaching variable.

Conditions

- c is constant with respect to x.

Limit sum law

$$\lim_{x \to a} [f(x) + g(x)] = L + M$$

Text version: The limit as x approaches a of f of x plus g of x equals L plus M.

When to use: Use this law to split the limit of a sum when both component limits exist.

Conditions

- The limit of f of x as x approaches a equals L.
- The limit of g of x as x approaches a equals M.
- L and M are finite real numbers for the form shown.

Limit product law

$$\lim_{x \to a} [f(x)g(x)] = LM$$

Text version: The limit as x approaches a of f of x times g of x equals L times M.

When to use: Use this law to multiply two component limits that both exist.

Conditions

- The limit of f of x as x approaches a equals L.
- The limit of g of x as x approaches a equals M.
- L and M are finite real numbers for the form shown.

Limit quotient law

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{L}{M}$$

Text version: The limit as x approaches a of f of x divided by g of x equals L divided by M.

When to use: Use this law to divide two component limits when the denominator limit is nonzero.

Conditions

- The limit of f of x as x approaches a equals L.
- The limit of g of x as x approaches a equals M.
- M is not zero.

Power limit law

$$\lim_{x \to a} [f(x)]^n = L^n$$

Text version: The limit as x approaches a of f of x to the power n equals L to the power n.

When to use: Use this law after establishing that f approaches L and the real-valued power is continuous at L.

Conditions

- The limit of f of x as x approaches a equals L.
- The power operation is real-valued and continuous at L.

Continuity and composition

Pass a limit through a continuous outer function after establishing the inner limit.

Continuous composition law

$$\lim_{x \to a} g(f(x)) = g(L)$$

Text version: The limit as x approaches a of g of f of x equals g of L.

When to use: Use this law when f approaches L and the outer function g is continuous at L.

Conditions

- The limit of f of x as x approaches a equals L.
- g is continuous at L.

Standard limits

Recognize the small-angle and exponential patterns that anchor many indeterminate limits.

Standard sine limit

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

Text version: The limit as x approaches zero of sine x divided by x equals one.

When to use: Use this standard limit when a trigonometric expression reduces to sine of a small angle divided by that angle.

Conditions

- Angles are measured in radians.
- The limit is two-sided over the real numbers.

Cosine difference limit

$$\lim_{x \to 0} \frac{1 - \cos x}{x} = 0$$

Text version: The limit as x approaches zero of one minus cosine x divided by x equals zero.

When to use: Use this standard limit for a first-power small-angle denominator; related squared-denominator forms have a different value.

Conditions

- Angles are measured in radians.
- The limit is two-sided over the real numbers.

Standard exponential limit

$$\lim_{x \to 0} \frac{e^x - 1}{x} = 1$$

Text version: The limit as x approaches zero of e to the x minus one divided by x equals one.

When to use: Use this limit when an exponential difference has a matching small increment in the denominator.

Conditions

- The base is the natural exponential e.
- The limit is two-sided over the real numbers.

Standard logarithm limit

$$\lim_{x \to 0} \frac{\ln(1+x)}{x} = 1$$

Text version: The limit as x approaches zero of the natural logarithm of one plus x divided by x equals one.

When to use: Use this limit when a natural logarithm approaches log of one and the denominator matches its small increment.

Conditions

- x is greater than negative one near the approach point.
- The logarithm is natural logarithm.

Rational end behavior

Compare leading powers when a rational function approaches positive or negative infinity.

Equal-degree rational end behavior

$$\lim_{x \to \pm\infty} \frac{a_n x^n + \cdots}{b_n x^n + \cdots} = \frac{a_n}{b_n}$$

Text version: For a rational function with equal numerator and denominator degree n, the limit at positive or negative infinity equals the ratio of the leading coefficients a sub n and b sub n.

When to use: Use this rule for polynomial ratios whose numerator and denominator have the same highest degree.

Conditions

- a sub n and b sub n are the nonzero leading coefficients.
- The numerator and denominator have the same finite degree n.