FORMULA REFERENCE

Integration Formulas

Identify the integrand pattern, preserve the constant of integration, and verify an antiderivative by differentiation.

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Basic antiderivatives

Reverse familiar derivative rules for powers, exponentials, reciprocals, sine, and cosine.

Power antiderivative

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C$$

Text version: The indefinite integral of x to the n with respect to x equals x to the n plus one divided by n plus one, plus a constant C.

When to use: Use this rule to integrate a power of x other than x to the negative one.

Conditions

- n is constant with respect to x.
- n is not negative one.
- Use an interval where the real-valued power is defined.

Natural exponential antiderivative

$$\int e^x \, dx = e^x + C$$

Text version: The indefinite integral of e to the x with respect to x equals e to the x plus a constant C.

When to use: Use this rule to integrate the natural exponential function.

Conditions

- C is an arbitrary constant of integration.

Reciprocal antiderivative

$$\int \frac{1}{x}\,dx = \ln|x| + C$$

Text version: The indefinite integral of one divided by x with respect to x equals the natural logarithm of the absolute value of x plus a constant C.

When to use: Use this rule for the reciprocal function on an interval that does not cross zero.

Conditions

- x is not zero.
- Choose an interval entirely on one side of zero.
- C is an arbitrary constant of integration.

Sine antiderivative

$$\int \sin x\,dx = -\cos x + C$$

Text version: The indefinite integral of sine x with respect to x equals negative cosine x plus a constant C.

When to use: Use this rule for sine of the integration variable; use substitution when the angle is a nontrivial inner function.

Conditions

- Angles are measured in radians.
- C is an arbitrary constant of integration.

Cosine antiderivative

$$\int \cos x\,dx = \sin x + C$$

Text version: The indefinite integral of cosine x with respect to x equals sine x plus a constant C.

When to use: Use this rule for cosine of the integration variable; use substitution when the angle is a nontrivial inner function.

Conditions

- Angles are measured in radians.
- C is an arbitrary constant of integration.

Linearity and methods

Separate sums and constants, or transform the integral when a direct rule does not fit.

Linearity of integration

$$\int [af(x) + bg(x)]\,dx = a\int f(x)\,dx + b\int g(x)\,dx$$

Text version: The integral of a times f of x plus b times g of x equals a times the integral of f plus b times the integral of g.

When to use: Use linearity to separate a sum and move constant factors outside the integral.

Conditions

- a and b are constant with respect to x.
- The displayed antiderivatives exist on a common interval.

Substitution rule

$$\int f(g(x))g'(x)\,dx = \int f(u)\,du, \quad u = g(x)$$

Text version: The integral of f of g of x times g prime of x with respect to x becomes the integral of f of u with respect to u, where u equals g of x.

When to use: Use substitution when the integrand contains a composition together with the derivative of its inner function, up to a constant factor.

Conditions

- g is differentiable on the interval of substitution.
- The substitution preserves the relevant interval and domain.

Integration by parts

$$\int u\,dv = uv - \int v\,du$$

Text version: The integral of u times d v equals u times v minus the integral of v times d u.

When to use: Use this identity when differentiating one factor simplifies it and the other factor can be integrated.

Conditions

- u and v are differentiable on the interval of interest.
- The displayed integrals exist on that interval.

Definite integrals

Use a verified antiderivative for endpoint evaluation or an average value on a closed interval.

Definite integral evaluation

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

Text version: The definite integral from a to b of f of x equals F of b minus F of a, where F prime equals f.

When to use: Use this result to evaluate a definite integral from a verified antiderivative.

Conditions

- f is continuous on the closed interval from a to b.
- F is an antiderivative of f on that interval.

Average value of a function

$$f_{\text{avg}} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

Text version: The average value of f on the interval from a to b equals one divided by b minus a times the definite integral of f from a to b.

When to use: Use this formula to find the continuous average of a function over a finite interval.

Conditions

- f is integrable on the closed interval from a to b.
- b is greater than a.